Distributions of First-passage and Thermodynamical Approach in Problems of Phase Sinchronization

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Abstract

The prototype problem of a pendulum connected to a heat bath with strong dissipation is common for many physical applications. The corresponding equations of motion describe the current through the Josephson junction, and phase drift in a synchronizing auto-generator, is related to the problems of "stochastic ratchets", to the motion of Brownian particles in a nonlinear system with anisotropic spatially periodic potential. The thermodynamic flux in such systems is related to the (random) particle escape time. In the present work the first-passage time distribution and its thermodynamic interpretation has been used to describe these phenomena. In particular, according to the example of auto-generator oscillating system, it is shown that the unperturbed average first-passage time is related to the value of the thermodynamic kinetic coefficient.

Keywords

First-passage Time; Phase Synchronization; Distribution with a Lifetime

Introduction

The first-passage time, which is exit out of the system of quasi-equilibrium-state under the influence of fluctuations (Stratonovich, 1963, 1967), is important in many physical phenomena. This value is necessary to know in the theory of phase transitions, by the study of the dynamics of complex molecules, the chemical reaction rate, in the calculation of the coefficient of surface diffusion in semiconductors, as well as in the analysis of failure of radio and optical tracking systems, automatic tracking. In (Pontryagin and Andronov, 1933) Pontryagin equations were obtained for the probability of achievement, which is a classic work of Kramers (1940). Various aspects of this issue have been considered in (Maier and Stein, 1993).

In (Stratonovich, 1995, Stratonovich and Chichigina, 1996, Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) it was shown that the probability of the lifetime can be described by exponential distribution and the expressions for the average lifetime are obtained. In (Stratonovich, 1995, Stratonovich and Chichigina, 1996, Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) a quasi-equilibrium Boltzmann distribution has been introduced (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) and it is shown that the entropy decreases in the measurement of the lifetime (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002). These results can be used to support the approach of (Ryazanov, 2004, 2006, 2011), in which the time to achievement of level is considered as thermodynamic variable. In contrast to (Stratonovich, 1963, 1967), a lifetime without impacts is separated, which is part of the generalized structure factor, and the value containing the impact on the system and thermodynamically conjugate to thermodynamic quantities of time to achieve level. The method of nonequilibrium statistical operator in (Ryazanov, 2001) is interpreted as an average of quasi-equilibrium statistical operator on the distribution of the lifetime of the system. In (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) a quasi-stationary distribution is obtained. Distribution obtained in (Ryazanov, 2004, 2006, 2011), after the integration over of lifetime is stationary and the possibility of exit out the system is taken into consideration. In contrast to the approach of (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) in (Ryazanov, 2004, 2006, 2011), the smallness in lifetime rather than the value, as well as the insignificant effects on random variable of lifetime is proposed. Therefore the condition that the average lifetime must be much larger correlation times, mandatory in (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002), not necessarily in the approach proposed in (Ryazanov, 2004, 2006, 2011). In this paper the results of (Ryazanov, 2004, 2006, 2011) in the fourth section shall be applied to physical problems described in the second section. In the third section, a

thermodynamic interpretation of lifetime is provided.

Phase Synchronization

In this section, the synchronization of the generator in the presence of noise is in consideration (Stratonovich, 1963, 1967, Tikhonov and Mironov, 1977). Effective means to achieve a more stable oscillation frequency is the external clock, that is, to supply voltage for the clock generator from another generator, which has a greater degree of frequency stability, while with a low power. An entrainment of the oscillation frequency and its stabilization are in it, however, the presence of fluctuations leads to a small deviation from the phase of its synchronous value, and to a large deviation of the phase, comparable to $\pi/2$, as well as the phase jumps by an integer number of periods. These jumps failing to be compensated, are accumulated and lead to the fact that the average frequency of oscillation does not coincide with the clock frequency, as well as to the fact that there is an irreversible phase diffusion, which gives the diffusion frequency instability.

If the intensity of the noise is small

$$K \ll \varepsilon \omega_c A_0^2 , \tag{1}$$

where ω_c is frequency of synchronization, the small and the relative variation of the amplitude

$$\left\langle (A - A_0^{\ 2}) \right\rangle << A_0^{\ 2} , \tag{2}$$

the exact value of the amplitude A incoming in the simplified equation for random phase φ can be replaced by an approximate value of A_0 and the equation for the phase of the form is obtained

$$\frac{d\varphi}{dt} = \Delta_0 - \Delta \sin\varphi - \frac{2\Delta\xi}{A_m} . \tag{3}$$

In (1)-(3) $\Delta_0 = (\omega_0^2 - \omega_c^2)/\omega_c \approx \omega_0 - \omega_c$ is the value of the initial mismatch of generators ω, and $\omega_0=(LC)^{-1/2}$ is resonant frequency of master clock, in which C is capacity, L is inductance, R is resistance of circuit of simplified diagrams of oscillator, M is electromotive force induced circuit due to the presence of inductive coupling, S is slope of cubic lamp characteristics, A is amplitude, A_m is amplitude of sync harmonic signal $A_m \cos\omega t$; while the formula $A_0 = 4(MS-RC)/M\gamma_1$ is stationary value of the amplitude, in which ratio γ_1 describes the extent of the cubic dependence, $\varepsilon=\omega_0^2(MS-RC)/\omega_c$, and K is correlation parameter of random forces ξ (intensity of the noise) from ratio of $<\xi\xi_\tau>=K\delta(\tau)$, $\Delta=\omega_c E/A_0$ which is band of synchronization (retention), $E=\omega_0^2 MCU_0$, U_0 is the amplitude of the sync voltage.

Equation (3) is applied if (1) and

$$E/A_0 \ll \varepsilon, \quad \varepsilon\omega_c >> \Delta_c, \tag{4}$$

$\Delta_c = \omega_c E/2A_0$, when linearization of the amplitude and is not allowed linearization of the phase. Equation (3) corresponds to the Fokker-Planck equation of the form

$$\frac{\partial\omega(\varphi)}{\partial t} = -\frac{\partial[(\Delta_0 - \Delta\sin\varphi)\omega(\varphi)]}{\partial\varphi} + \frac{K}{2A_0^{\ 2}}\frac{\partial^2\omega(\varphi)}{\partial\varphi^2} , \tag{5}$$

where $\omega(\varphi)$ is probability density of the distributions of the random phase values φ. Note that equation (3) and (5) arise in many physical problems. Thus, in (Feigel'man and Tsvelik, 1982) this equation (with other symbols and, respectively, with different physical meaning of the parameters) describes the motion placed in a thermal reservoir pendulum with strong dissipation under the action of gravity and torque. In (Ivanchenko and Zil'berman, 1969, Ambegaonar and Halperin, 1968) this equation describes the current through the Josephson junction with a large dissipation. In addition to maintenance phase of clock oscillator (Stratonovich, 1963, 1967) it describes the care phase of the laser (Haken, 1980). In recent years increasing attention of researchers has been paid to the so-called "stochastic ratchets" (Magnasco, 1993) under the understanding of which is the motion of Brownian particles in a nonlinear system with a spatially periodic anisotropic potential. In such systems, when exposed to different types of fluctuations, it may cause the probability flux along the spatial coordinates. In (Postnov et al. 1996) the mathematical model of the radio system phase-locked with an appropriate choice of parameters is reduced to one-dimensional model with a spatially periodic sawtooth potential. Experimental results in a one-dimensional approximation showed the effect of emergence of a probability flux. In this case two different mechanisms for its occurrence have been implemented. The experimental setup corresponding to a non-zero probability flux drift mutual phases of the two oscillators, the direction and magnitude are determined by the parameters having affect on the installation of signals.

Stationary solution of (5) is written in (Stratonovich, 1963, 1967, Tikhonov and Mironov, 1977) as

$$\omega_{st}(\varphi) = N^{-1}\exp\{D_0\varphi + D\cos\varphi\}\int_{\varphi}^{\varphi+2\pi}\exp\{-D_0\psi - D\cos\psi\}d\psi \tag{6}$$

$N=4\pi^2 exp\{-\pi D_0\}/|I_{iD0}(D)|^2$, where $I_{iD0}(D)$ is tabulated Bessel function with imaginary argument and imaginary index, $D_0=A_m^2/\Delta K$, $D_0/D=\Delta_0/\Delta$. The parameter D characterizes the signal to noise ratio in the band of synchronization, and D_0 is the value of the relative initial detuning. In (Tikhonov and Mironov, 1977) the value

$$J_{(\pm)} = N^{(\pm)}{}_1 = \frac{\Delta \exp\{(\pm)\pi D_0\}}{4\pi^2 D\left|I_{iD_0}(D)\right|^2} \tag{7}$$

is introduced. That is average number of jumps of the phase difference by 2π per unit time in the direction of its increase (decrease). The value $J=G_1=N_1^+-N_1^-=\Delta sh(\pi D_0)/2\pi^2 D\ |I_{iD0}(D)|^2=\Delta_0 sh(\pi D_0)/2\pi^2 D_0|I_{iD0}(D)|^2$ characterizes the average flow, and the value $N_1=N_1^++N_1^-=cth(\pi D_0)G_1$ – the average diffusion. In (Stratonovich, 1963, 1967, Tikhonov and Mironov, 1977) it was defined as the times of the transition of phase from the vicinity of a stable equilibrium state in the neighboring states of stable equilibrium. So for the average time T_1 of first passage, the nearest stable equilibrium states from the initial steady state in (Stratonovich, 1963, 1967) the expression are written

$$T_1 = \frac{4\pi^2 D\exp\{\pi D_0\}\left|I_{iD_0}(D)\right|^2}{\Delta(1+\exp\{2\pi D_0\})} = \frac{sh(\pi D_0)}{ch(\pi D_0)J} = \frac{1}{N_1}; \quad T_0 = T_{1|D_0=0} = \frac{4\pi^2 D\left|I_{i0}(D)\right|^2}{2\Delta}, \tag{8}$$

T_0 is mean time to reach the boundary in the equilibrium state.

A Thermodynamic Approach to the Description of the Jumps of the Phase Difference.

It should distinguish between the different meanings of "flow." Thus, there is a "probability flux" out of this potential well into the next, as it exists in the equilibrium and in a non-equilibrium state. It is also associated with the time of reach of the boundary. For example, in the equilibrium state it is (average) flow J_0 and the mean time to reach the boundary $T_0=2/J_0=2/N_0$ (Magnasco, 1993) (symmetrical with respect to the left and right outputs). In the non-equilibrium state (at nonzero pumping D_0) it is frequency hopping (flows) to the left and to the right J_+ $(=N_1^+)$ and J_- $(=N_1^-)$. And then there will be an uncompensated flow $J= J_+- J_-$ $(G_1=N_1^+-N_1^-)$. It is this flow (and not J_+) that is a thermodynamic quantity. The relationship between the thermodynamic flow J and J+, J- can be dependent on the magnitude with different degree of impact. For example, if the difference between the depths of the wells is small flows themselves J_+, J_- can be large, and $J << J_+$. Conversely, if the system is highly non-equilibrium, then conversely $J_- << J_+$ and $J \sim J_+$. The flow J_0 (or the time to reach T_0) is, therefore, an inherent characteristic of the system, a kind of potential to respond the force. It seems to be similar to the kinetic coefficient. You also need to find out what time scale is in consideration. The consideration with streaming necessarily requires that the time scale has been much more time taken by the particle in the well (only then can we even talk about of the flow). Otherwise, the flow doesn't exist as statistical value. Thus, the formula for the frequency hopping in equilibrium is obtained by setting $D_0= 0$ in (7)

$$J_0 = N_0 = \frac{\Delta}{4\pi^2 D\left|I_0(D)\right|^2}, \tag{9}$$

this "equilibrium" frequency of hopping. Power is indicated here as D_0, and D is parameter such as temperature. Now the same expression (7) in the presence of power should be paid attention to find uncompensated flow $J = N_1^+-N_1^-$. Note that for small forces Bessel function is almost the same as I_0 (compared to the contribution of the leading members).

So, it can be written

$$J \sim (exp\{\pi D_0\} - exp\{-\pi D_0\})\, J_0, \tag{10}$$

that is, the thermodynamic flux J is expressed through "internal characterization" J_0. In the spirit of linear thermodynamics the exponential must be extended in (10) and left linear member the total acquisition is that

$$J \approx X\, J_0 = 2X / T_0, \tag{11}$$

X is the power (in this case $2\pi D_0$), and the coefficient of proportionality is related to the achieved time. That is (equilibrium) first-passage time similar to the kinetic coefficient. Since the results are utilized (for synchronizing the generator or for a nonlinear oscillator), it can be said that it is confirmation of the theory (Ryazanov, 2004, 2006, 2011) on the microscopic level.

Heuristically (without specifying the model) the following consideration can be cited the general formula for the internal production of entropy is expressed as

$\delta S = J\, X.$

As in conventional linear thermodynamics, X can be expressed from (11), and we have

$$\delta S = J^2 T_0 /2. \tag{12}$$

The positivity rate of entropy provides a positive lifetime. Thus, the unperturbed mean time is to reach the boundary maps to the kinetic coefficient in the approximation of linear nonequilibrium thermodynamics.

Distribution with a Lifetime

In (Stratonovich, 1963, 1967) the distribution is obtained

$$\omega(\varphi,\tau)=\omega_1(\varphi)\exp\{-\Lambda\tau\},\quad \Lambda=\langle\tau\rangle^{-1}, \tag{13}$$

(where τ is time to reach the absorbing boundary), the corresponding distribution of (14) obtained in (Ryazanov, 2004, 2006, 2011). The value of τ in this case serves as a lifetime (Stratonovich, 1963, 1967), and Λ plays the role in the conjugate of the thermodynamic quantity. Distribution (13) in (Stratonovich, 1963, 1967) corresponds to the "stable decrease" of the number of remaining in stationary area of the phase space of the representative points. In (Stratonovich, 1963, 1967) the value Λ is expressed in terms of the intensity of jumps of phase difference (7) and the average lifetime, $\Lambda=\gamma=Jch(\pi D_0)/sh(\pi D_0)$, where J is average flow of the second and third Sections, and equating $\Lambda=\gamma$ is based on equating the expressions (13) and (14) a different expression for γ is obtained that does not use these assumptions.

In (Ryazanov, 2004, 2006, 2011) the distribution containing lifetime (instead of τ in (13)) is introduced. The joint probability density for the energy values of E and the lifetime is

$$p(E,\Gamma)=\frac{e^{-\beta E-\gamma\Gamma}\omega(E,\Gamma)}{Z(\beta,\gamma)};$$
$$Z(\beta,\gamma)=\int e^{-\beta E-\gamma\Gamma}dz=\iint dEd\Gamma\omega(E,\Gamma)e^{-\beta E-\gamma\Gamma} \tag{14}$$

where z is dynamic variables ($z=q_1,\dots,q_N,\ p_1,\dots,p_N$), $\omega(E,\Gamma)$ is generalized structure factor, the number of phase points between $E,E+dE$; $\Gamma,\Gamma+d\Gamma$ equals $\omega(E,\Gamma)\ dEd\Gamma$ (Ryazanov, 2004, 2006, 2011, 2005, 2012).

Integrating (14) by Γ , the stationary distribution for E of the form is acquired

$$p(E)=\int P(E,\Gamma)d\Gamma=\frac{e^{-\beta E}}{Z(\beta,\gamma)}\int_0^\infty e^{-\gamma\Gamma}\omega(E,\Gamma)d\Gamma. \tag{15}$$

A similar result was obtained in (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002). However, in (Chichigina and Netrebko, 2001, Chichigina, 1999, 2002), the distribution of the form (13) is obtained, which is, in general, not stationary. In (Ryazanov, 2004, 2006, 2011) the generalized structure factor gives meaning to the stationary joint probability density for the E and Γ. We write

$$\omega(E,\Gamma)=\omega(E)\omega_1(E,\Gamma)=\omega(E)\sum_{k=1}^n R_k f_k(\Gamma,E). \tag{16}$$

In the expression (16) it is assumed that the system has n classes of ergodic states; in which R_k is the probability of the system in k-th class of ergodic states, and $f_k(\Gamma,E)$ is density of lifetime Γ distribution in this class of ergodic states. As a physical example of such a situation (characteristic of metals, glasses) the potential functions of many complex systems can be specified (Olemskoi, 1993), including the potential of the system discussed in Section 2. Potential minima correspond to metastable phases, disordered structures, etc. In such systems the phase space is divided into isolated regions, each of which corresponds to a metastable thermodynamic state, and the number of these regions increases exponentially with increase in the total number of particles (or quasiparticles) (Edwards, 1994). Quasithermodynamic theory of structural transformation of the alloy Pd-Ta-H, based on this model, has been developed in (Avdyukhina et al. 2002). Expression (16) is applicable to the description of such systems. the explicit form of the distribution f_k is defined in (16), namely the gamma distribution

$$f_k(x)=\frac{1}{\Gamma(\alpha_k)}\frac{1}{b_k^{\alpha_k}}x^{\alpha_k-1}e^{-x/b_k},\quad x>0,\qquad f_k(x)=0;\quad x<0;$$
$$\int_0^\infty e^{-\gamma_k x}f_k(x)dx=(1+\gamma_k b_k)^{-\alpha_k} \tag{17}$$

((α) is gamma function, b_k are parameters of distribution $f_k(\Gamma=x)$). Substituting (17) in (15) - (16), we obtain

$$p(E)=\frac{e^{-\beta E}}{Z(\beta,\gamma)}\omega(E)\int_0^\infty\sum_{k=1}^n R_k f_k(\Gamma,E)e^{-\gamma_k\Gamma}d\Gamma=\frac{e^{-\beta E}}{Z(\beta,\gamma)}\omega(E)\sum_{k=1}^n R_k(1+\gamma_k b_k)^{-\alpha_k}. \tag{18}$$

From (14) and the fact that $\partial lnZ/\partial\gamma_{/\beta}=-\langle\Gamma\rangle$:

$$\langle\Gamma_\gamma\rangle_\kappa=\frac{\alpha_k b_k}{1+\gamma_k b_k};\qquad \Gamma_{0\kappa}=\langle\Gamma_\gamma\rangle_{\kappa/\gamma=0}=\alpha_\kappa b_\kappa;$$
$$1+\gamma_\kappa b_\kappa=\frac{\Gamma_{0k}}{\langle\Gamma_\gamma\rangle_k};$$

$$(1+\gamma_k b_k)^{-\alpha_k}=exp\{-\alpha_k ln(\frac{\Gamma_{0k}}{\langle\Gamma_\gamma\rangle_k})\}. \tag{19}$$

In many cases, instead of the gamma distribution (17) exponential distribution can be used with $\alpha_k=1$. When $\alpha_k=1$ and $b_\kappa=\Gamma_{0\kappa}$ in (Ryazanov, 2005, 2012) the distribution is obtained

$$p(E) = \frac{e^{-\beta E}}{Z(\beta,\gamma)}\omega(E)\sum_{k=1}^{n} R_k \frac{\langle \Gamma_\gamma\rangle}{\langle \Gamma_0\rangle} = \frac{e^{-\beta E}}{Z(\beta,\gamma)}\omega(E)\sum_{k=1}^{n} R_k \left[1+\frac{\gamma_k}{\lambda_k}\exp\{(\beta PV)_{0k}\}\right]^{-1}$$

where P_k, V_k, T_k are pressure, volume and temperature, respectively, in the k-th class of metastable states, and index 0 at the bottom indicates the absence of disturbances, when $\gamma_k=0$, $\lambda_k=\lambda$ is intensity of the flow of energy in the system (subsystem), which is equal to the dynamic equilibrium of the output intensity. This distribution tends to Gibbs at $\gamma\to 0$. For large values of γ, $p(E)\to 0$. At $\gamma\approx\lambda$,

$$p(E)\sim\frac{e^{-\beta E}}{Z(\beta,\gamma)}\omega(E)\sum_{k=1}^{n} R_k \mathrm{e}^{-(\beta P)_{0k} E v_k/u}, \quad v_k=V_k/V,\ u=E/V.$$

As a physical example of the value γ/λ consider its value for the random generator synchronized phase (Section 2), where (Stratonovich, 1963, 1967, Tikhonov and Mironov, 1977), average time to reach (in the approximation of the initial steady state) is defined by (8), where the value of D_0 is related to disturbance γ, $\langle\Gamma_\gamma\rangle=T_1$ (13), $\Gamma_0=T_0=1/2\lambda$, $\lambda = N_0 = \dfrac{\Delta}{4\pi^2 D|I_0(D)|^2}$ is the intensity of the phase jumps at $D_0=0$ (9). From (19) at $\alpha=1$, $\Gamma_0=b$,. $\gamma=[\dfrac{1}{\langle\Gamma_\gamma\rangle}-\dfrac{1}{\langle\Gamma_0\rangle}]$. Then

$$\frac{\gamma}{\lambda}=\frac{1}{\lambda}\left[\frac{1}{\langle\Gamma_\gamma\rangle}-\frac{1}{\langle\Gamma_0\rangle}\right]=\frac{[1+\exp\{2\pi D_0\}]|I_0(D)|^2}{\exp\{\pi D_0\}|I_{iD_0}(D)|^2-2}=$$

$$\frac{\pi D_0[\exp\{\pi D_0\}+\exp\{-\pi D_0\}]}{sh(\pi D_0)\{1+2D_0^2 I_0^{-2}(D)\sum_{n=1}^{\infty}(-1)^n I_n(D)/(n^2+D_0^2)\}-2}. \tag{20}$$

This quantity tends to 0 by $D_0\to 0$ and to ∞ as D_0 by $D_0\to\infty$ (although D_0 takes limited value). As the value γ (20) is known, the distribution of the form (14), (15) can be used to solve all sorts of non-equilibrium problems, for example, for the thermodynamic description of the jumps of the phase difference (Section 3).

Conclusion

Thus, the use of the lifetime as a thermodynamic variable allows us to obtain a stationary nonequilibrium distribution for "primary" variable energy (in our example - the random phase). These distributions can not only describe in detail the processes of phase synchronization, but also a number of other phenomena mentioned in the Introduction.

Comments have been made on the time scale and the behavior of the average time to reach the borders has been qualitatively discussed. Relative to the average time of exit out of the potential well, there are two different time scales. Quasi-equilibrium in a potential well is not the same as the general equilibrium when the average is taken over the set of potential wells (or periods of employment in terms of queuing theory). The exit time is large for a number of systems with small fluctuations, and these two scale are the same. However, in small systems (or systems with large flows), this is not so. Ordinary thermodynamics, Boltzmann equation and thermodynamic equation of Maxwell describe great times together with the assumption that exit time is also very large. In the case where it is essential to record the value of periods of employment, it is not – here for all the time dynamics lying on a small time scale, and a quasi-equilibrium is established on time scales shorter than the output from the potential well, while the global equilibrium is established for much larger lifetimes. Therefore, in such situations, it is difficult to find evolution equations. The dependence on memory can be on a scale larger than the time to reach. As in (Stratonovich, 1963, 1967, Tikhonov and Mironov, 1977, Stratonovich, 1995, Stratonovich, Chichigina, 1996, Chichigina and Netrebko, 2001, Chichigina, 1999, 2002), one can imagine the process of hopping between potential wells as Brownian motion, but the memory may be housed in the complex structure of individual well. This dependence of the memory can be neglected in the simplest cases. Accounting random time spent in the potential well leads to a semi-Markov processes (if these times of stay are not exponentially distributed). The assumption of exponential distribution (Stratonovich, 1963, 1967, Stratonovich, 1995, Stratonovich, Chichigina, 1996, Chichigina and Netrebko, 2001, Chichigina, 1999, 2002) leads to the Markov nature of the process. In the general case the distribution for the first passage time can be written with the dependence on the initial position. These distributions will be more complex than the exponential distribution, and the result will be a semi-Markov process, that is, it already has a memory. The average time to reach is almost constantly in the potential well, but it changes abruptly near its boundaries. If the potential well is deep enough, the corrections are of higher order. However at high temperatures or at high thermodynamic forces, the amendment on the borders can be compared with the

behavior in the depths of the pit.

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